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Sensitivity Analysis Techniques Applied to a System of Hyperbolic Conservation Laws, Part II

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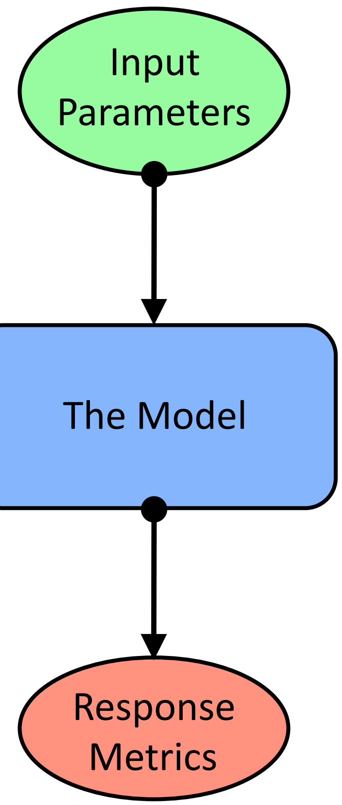
The Sensitivity Analysis Story

- Our problem of interest has 4 inputs and 8 outputs
- Outputs (responses) have different properties:
  - monotonic vs. non-monotonic
  - smooth vs. discontinuous
  - noisy vs. clean
- We examine different SA techniques:

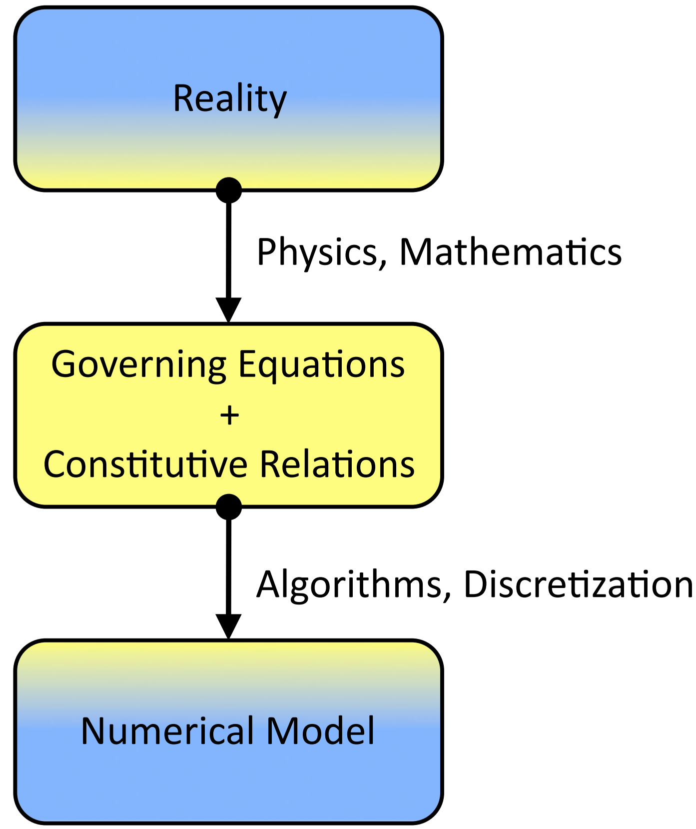
  LHS, LP-Tau } sampling
  PCE } stochastic expansion
  SDP, ACOSSO, DACE } surrogates

- We compute sensitivity indices and compare them to exact values; in particular, we examine performance with respect to sampling resolution



The Application Space Story

- Hyperbolic Conservation Laws are PDEs that describe the conservation of mass, momentum, and energy.
  - Constitutive relations describing specific materials are also required.
  - This combination is a mathematical model of reality.
- We use algorithms to obtain discrete equations from the mathematical model, and solve the discrete equations using a computer.
  - Such simulations provide approximate numerical solutions to the mathematical model.



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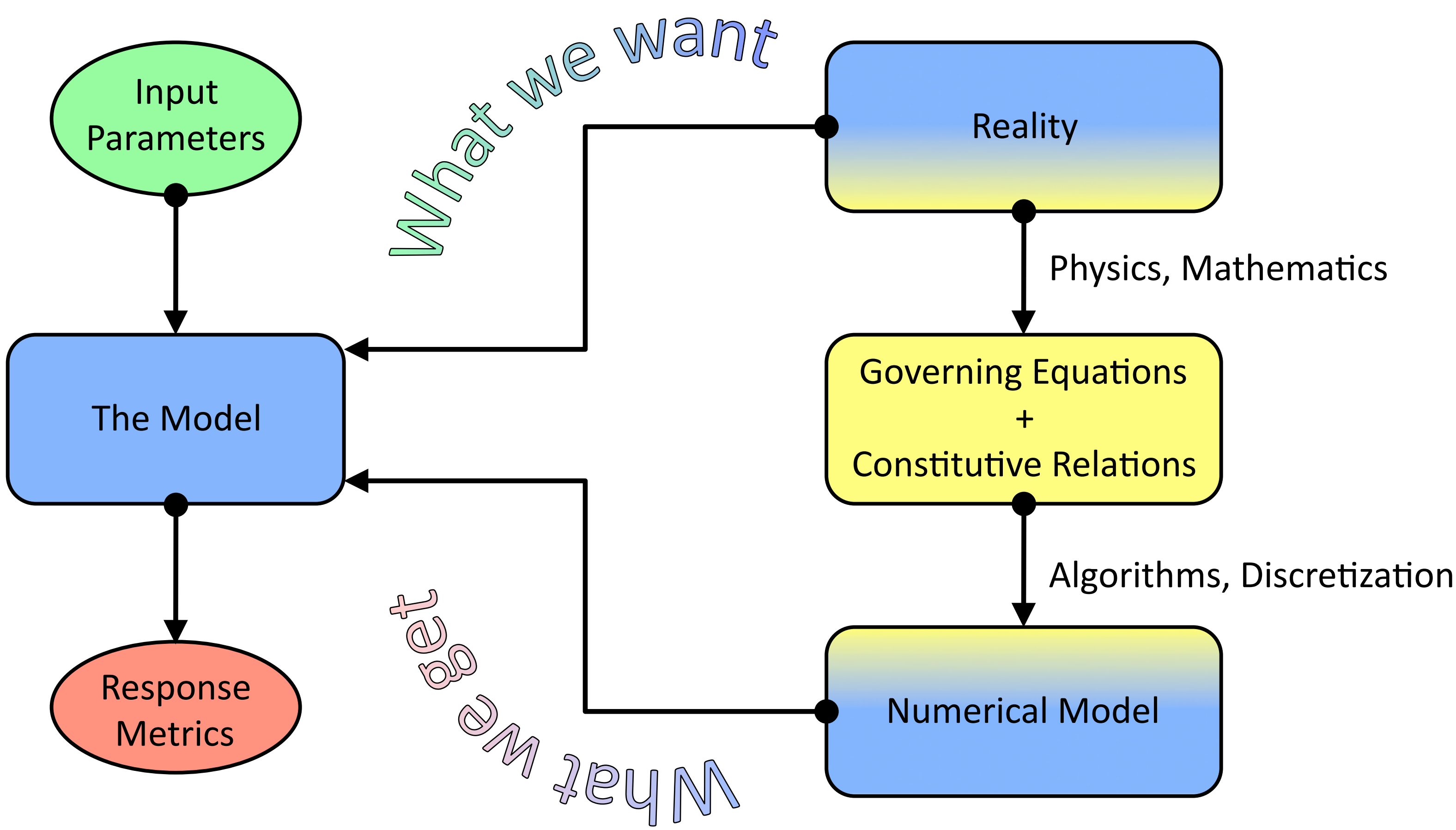
What we get from Sensitivity Analysis of Computer Simulations



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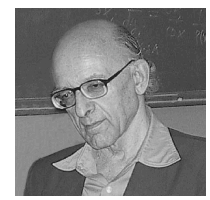
Correlation and Variance-Based Decomposition (VBD) are global sensitivity characterizations of uncertainty in model outputs Y.

- Goal: to assess inputs over a hypercube of interest.
- Correlation analysis identifies the strength and direction of a *linear* relationship between input and output.
- VBD identifies the fraction of the variance in the output that can be attributed to an individual variable alone or with interaction effects.
  - Main effect sensitivity S_i is the fraction of the uncertainty in Y that can be attributed to input x_i *alone*
  - Total effect index T_i is the fraction of the uncertainty in Y that can be attributed to x_i *and its interactions with other variables*
  - Calculation of S_i and T_i requires the evaluation of m-dimensional integrals, approximated by Monte-Carlo sampling.
  - *Computationally intensive*, as replicated sets of samples are evaluated: N samples and D inputs → evaluation of $N \times (D+2)$ samples.

$$S_i = \frac{\mathrm{Var}_{x_i}\left[E(Y|x_i)\right]}{\mathrm{Var}(Y)}$$

$$T_i = \frac{E\left[\mathrm{Var}(Y|x_{-i})\right]}{\mathrm{Var}(Y)}$$

$$x_{-i} \equiv (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_m)$$



I.M. Sobol' developed these ideas

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Generalized Polynomial Chaos Expansions approximate the response with a spectral projection using orthogonal polynomial basis functions.

- Expand the response R in terms of prescribed basis functions ψ_j:

$$R = \sum_{n=0}^{N} \alpha_n \psi_n(\xi) \quad \text{such that} \quad \begin{aligned} \Psi_0(\xi) &= \psi_0(\xi_1)\,\psi_0(\xi_2) = 1 \\ \Psi_1(\xi) &= \psi_1(\xi_1)\,\psi_0(\xi_2) = \xi_1 \\ \Psi_2(\xi) &= \psi_0(\xi_1)\,\psi_1(\xi_2) = \xi_2 \\ \Psi_3(\xi) &= \psi_2(\xi_1)\,\psi_0(\xi_2) = \xi_1^2 - 1 \\ \Psi_4(\xi) &= \psi_1(\xi_1)\,\psi_1(\xi_2) = \xi_1\xi_2 \\ \Psi_5(\xi) &= \psi_0(\xi_1)\,\psi_2(\xi_2) = \xi_2^2 - 1 \end{aligned} \quad \text{etc.}$$

  - The basis functions are orthogonal wrt some weight function
  - The coefficients α_n are fit to the data
- This approach is *nonintrusive* by estimating the coefficients α_n using:
  - Sampling (expectation)
  - Point collocation (regression)
  - Tensor-product quadrature
  - Smolyak sparse grid quadrature
- Wiener-Askey Generalized PCE is an "optimal" form of this method.
  - *Key idea*: use a set of basis functions $\psi_n(\xi)$ that are related to the assumed underlying distribution, leading to exponential convergence
  - E.g., the set of Legendre polynomials $P_n(\xi)$, orthogonal on [-1,1] with weight function unity, are the optimal basis for a uniform distribution

Other response surface models provide alternatives to sampling-based approaches.

- SDP = <u>S</u>tate-<u>D</u>ependent <u>P</u>arameter Regression
  - SDP modeling* is a class of non-parametric smoothing, first suggested by Young§, that is similar to smoothing splines and kernel regression approaches but is performed using recursive (non-numerical) Kalman filter and associated fixed interval smoothing.
  - Good for additive models, and flexibile in adapting to local discontinuities, strong non-linearity, and heteroskedasticity.
- ACOSSO = <u>A</u>daptive <u>CO</u>mponent <u>S</u>election and <u>S</u>moothing <u>O</u>perator
  - ACOSSO† is a multivariate smoothing-spline approach (COSSO‡) that is augmented by a weighted (w_j), scaled (λ) penalty function:

$$\hat{f} = \min_{f \in \mathcal{F}} \left\{ \frac{1}{N} \sum_{i=1}^{N} \left(Y_i - f(x_i) \right)^2 + \lambda \sum_{d=1}^{D} w_d \left\| P^d f \right\| \right\}$$

D = # inputs

  - ACOSSO is thought to perform best for a reasonably smooth underlying response.
- DACE = <u>D</u>esign and <u>A</u>nalysis of <u>C</u>omputer <u>E</u>xperiments
  - Gaussian Process emulator for the data

* Ratto, M., Pagano, A., Young, P. C., "State dependent parameter meta-modelling and sensitivity analysis," *Comput. Phys. Comm.,* **177**, pp. 863–876 (2007).

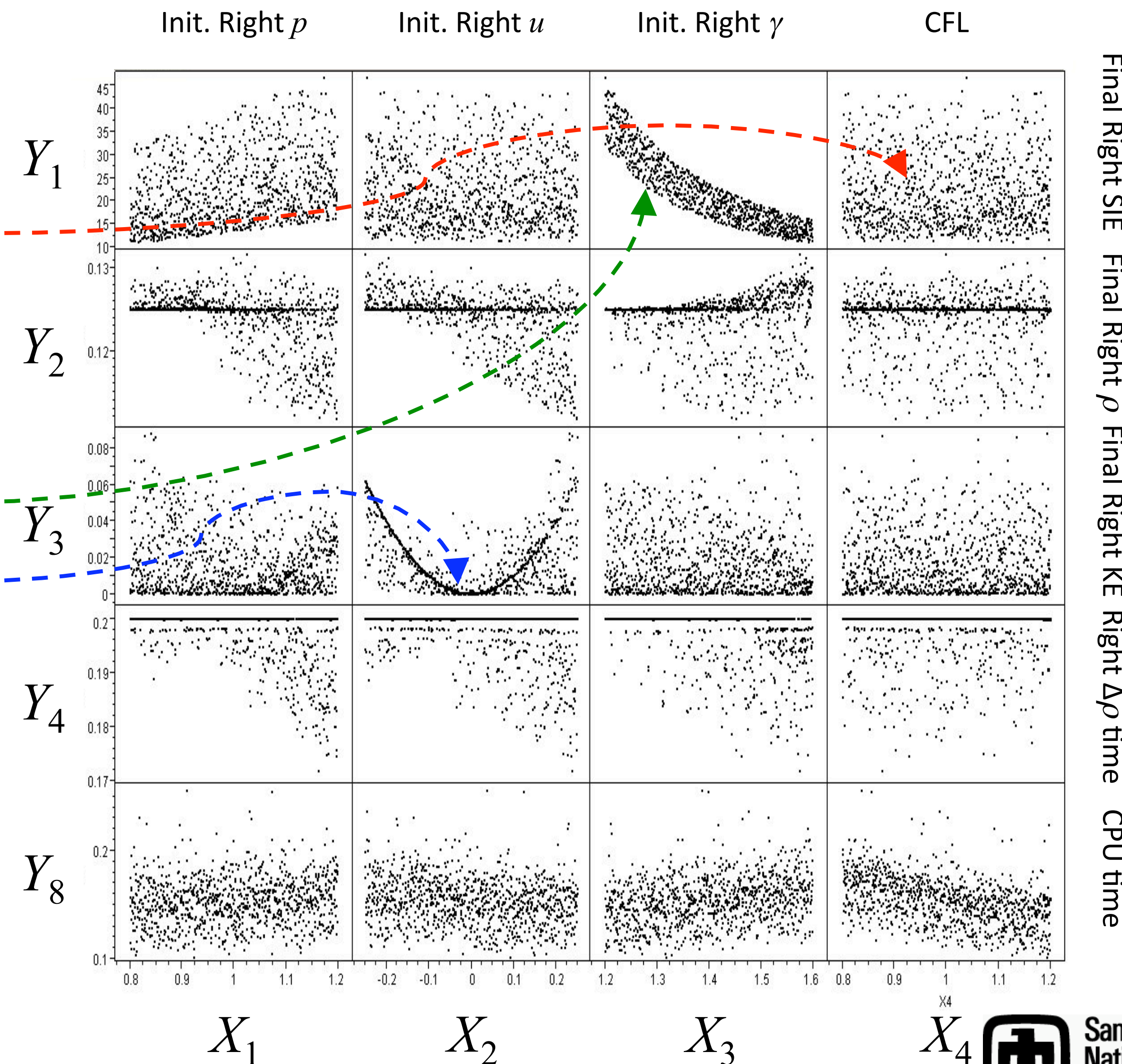
§ Young, P. C. "The identification and estimation of nonlinear stochastic systems," in *Nonlinear Dynamics and Statistics*, A. I. Mees et al., eds., Birkhauser, Boston (2001).

† Storlie, C.B., Bondell, H.D., Reich, B.J., Zhang, H.H., "Surface estimation, variable selection, and the nonparametric oracle property," *Stat. Sinica,* to appear (2010).

‡ Y. Lin,Y., and H. Zhang, H., "Component selection and smoothing in smoothing spline analysis of variance models," Ann. Stat., **34**, pp. 2272–2297 (2006).

Scatterplots of model outputs give some insights into the distributions.

- Some outputs appear insensitive:
  - Y_1 variation with X_4
- Some trends in the data are clear:
  - Y_1 variation with X_3
  - Y_3 variation with X_2
- VBD sensitivity indices quantify this behavior…



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We show results for estimators of the main and total sensitivity indices S and T for several methods.

Meta-models

- DACE 256 Gaussian process approach, 256 samples
- ACOSSO 256 adaptive smoothing spline, 256 samples
- SDP 256 non-parametric smoothing, 256 samples

Analytic VBD

- PCE6 1296 analytic VBD, 6th-order, uniform distr., 1296 samples
- JRC 196k 196k sample, Soboll´/Saltelli estimates *The usual "gold standard"*
- PCE4 256 analytic VBD, 4th-order, uniform distr., 256 samples

LHS Sampling

- LHS 60000 6.e+4 samples, LHS sampling-based VBD
- LHS 6000 6.e+3 samples, LHS sampling-based VBD

Full Factorial

- A-EXACT 160k 1.60e+5 (=20^4) ALEGRA samples, F.F. VBD
- A-EXACT 2.56M 2.56e+6 (=40^4) ALEGRA samples, F.F. VBD
- R-EXACT 160k 1.60e+5 Riemann (exact) samples, F.F. VBD
- R-EXACT-2.56M 2.56e+6 Riemann (exact) samples, F.F. VBD

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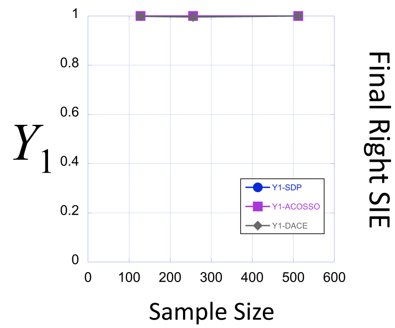
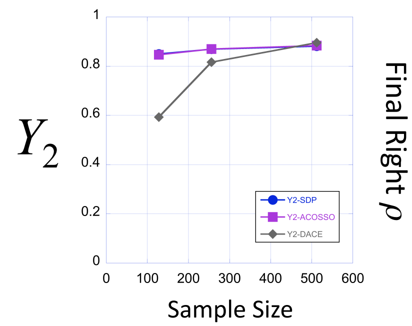
These analyses present several questions.

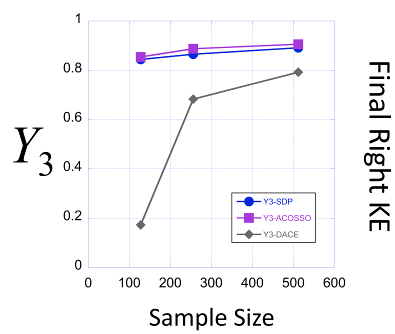
- *Do these approaches give consistent results, e.g., for rankings?*
- *Do these results vary for the different outputs,* Y_1–Y_4, Y_8*?*
- *How to these results depend on the different inputs,* X_1–X_4*?*
- *Do these results "converge"?*
- *How to sampling and meta-model results compare?*
- *Can we distinguish among different meta-models?*
- *How to exact solution results compare to ALEGRA results?*

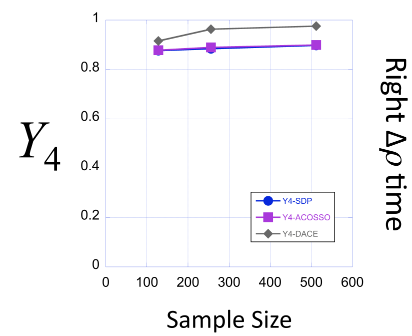
Comparison of R^2 for different meta-models under sample size gives a measure of the goodness-of-fit.

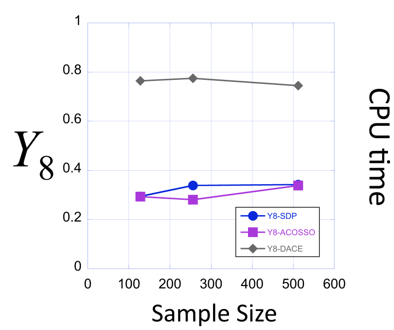
- The R^2 statistic is plotted for SDP, ACOSSO, and DACE (GP) emulators built with sample sizes: N=128, 256, 512.
- The goodness-of-fit clearly varies with the output:
  - Y_1 is very well fit
  - Y_2, Y_4 are reasonably well fit
  - Y_3 is reasonable with SDP, ACOSSO, but not so well with DACE (GP)
  - Y_8 is fit consistently better with DACE than the consistently poor fit with SDP and ACOSSO







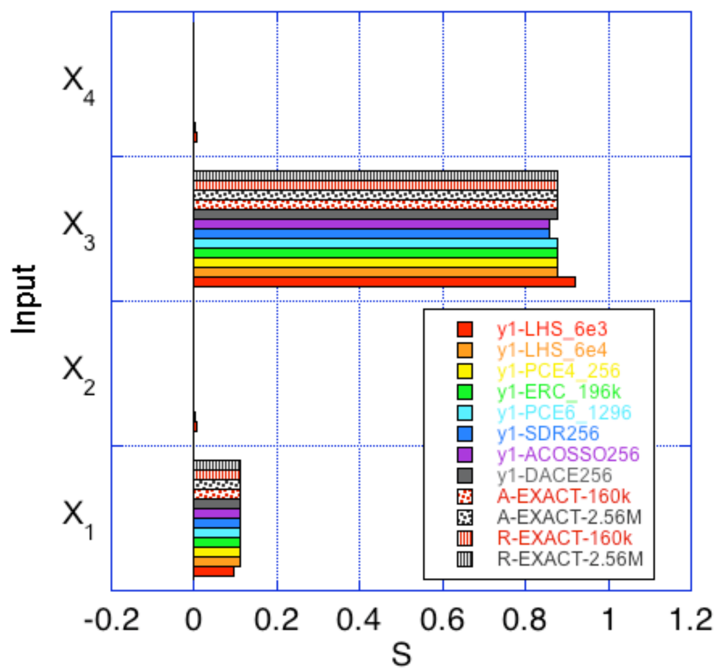




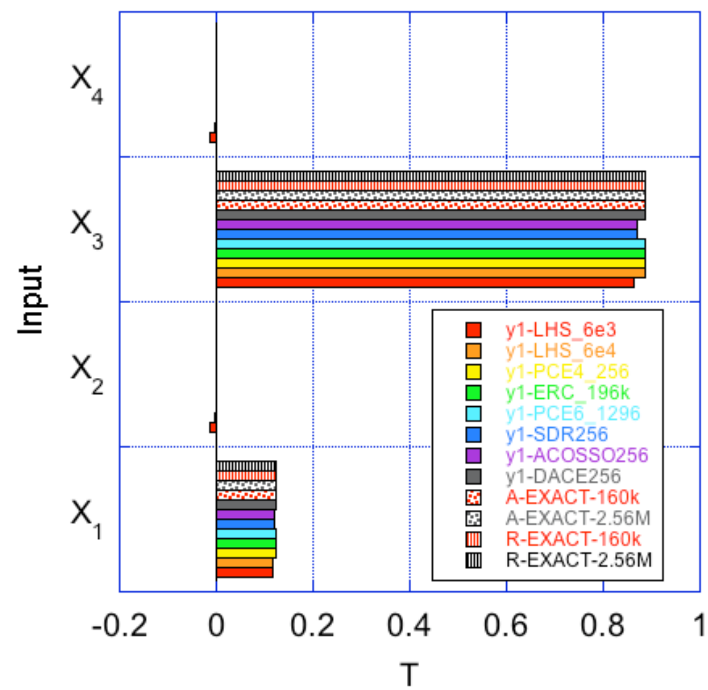
The sensitivity indices S and T for Y_1 perform similarly for all approaches.

- As anticipated, Y_1 (SIE) depends strongly on X_1 (p_R) and X_3 (γ_R)
- Sampling, meta-model, and "exact" results are all consistent.

Main



Total

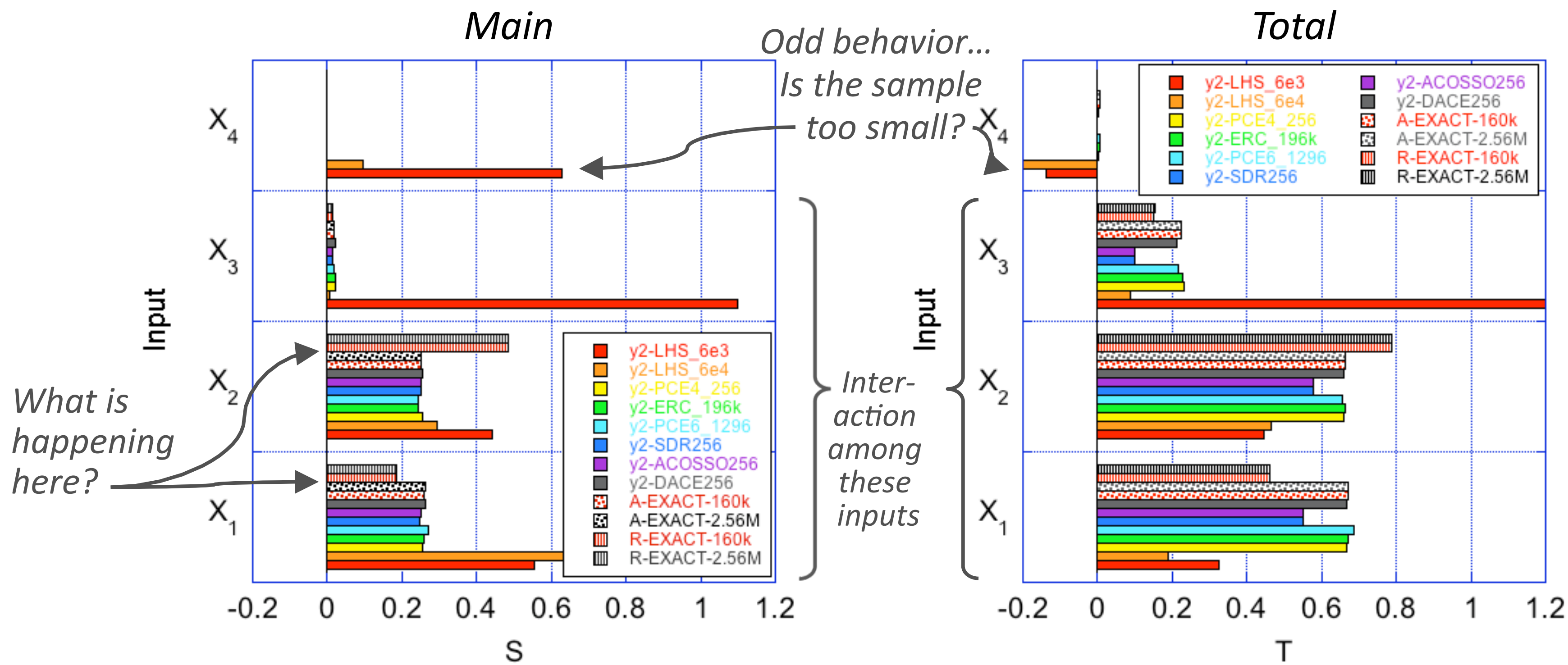


LHS 6000 PCE4 256 PCE6 1296 ACOSSO 256 A-EXACT 160k R-EXACT 160k

LHS 60000 JRC 196k SDP 256 DACE 256 A-EXACT-2.56M R-EXACT-2.56M

The sensitivity indices for Y_2 have some unusual features.

- For Y_2 (final right ρ), LHS has different ranking, particularly for 6.e+3 samples and esp. wrt X_3 (γ_R) and X_4 (CFL).

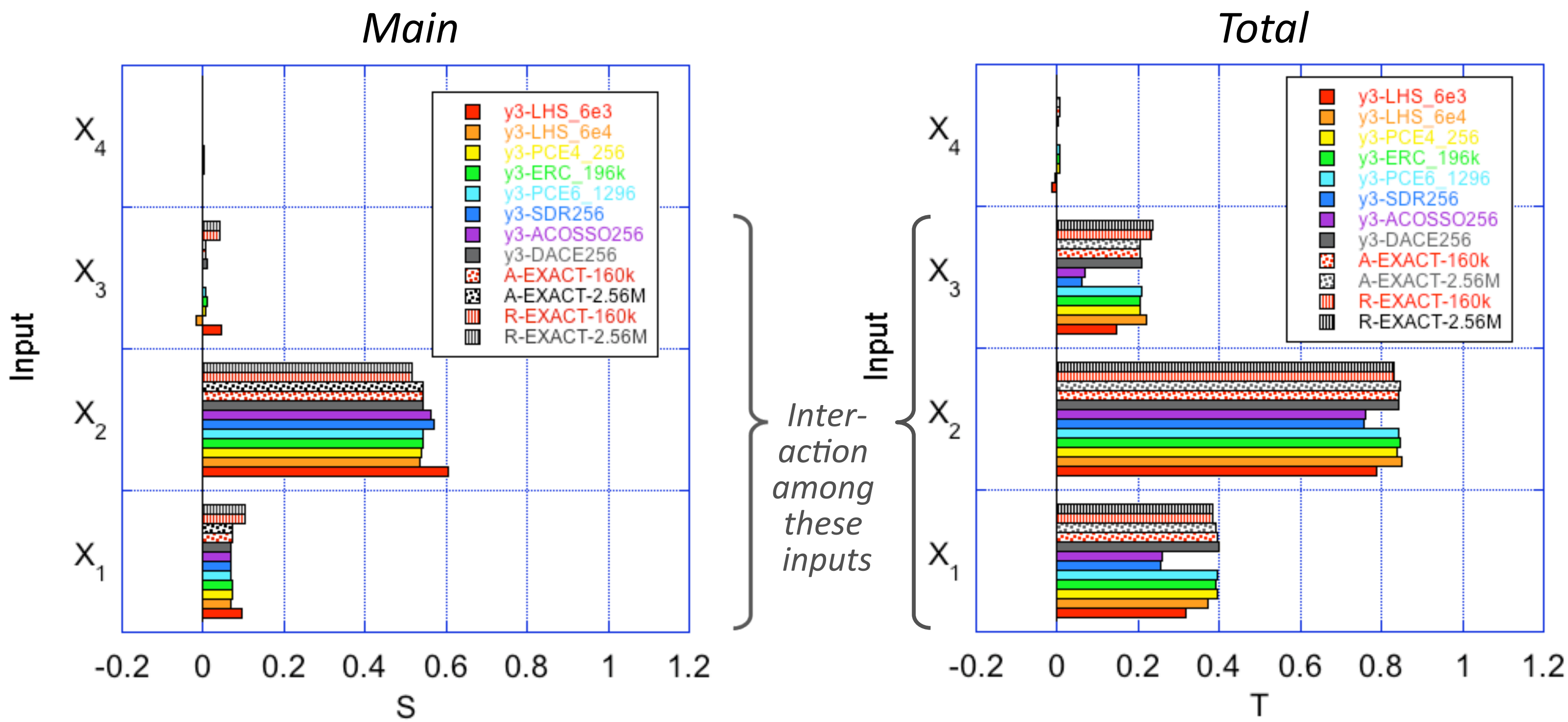


LHS 6000 PCE4 256 PCE6 1296 ACOSSO 256 A-EXACT 160k R-EXACT 160k

LHS 60000 JRC 196k SDP 256 DACE 256 A-EXACT-2.56M R-EXACT-2.56M

The sensitivity indices for Y_3 perform similarly for all approaches.

- As anticipated, Y_3 (final right KE) depends strongly on X_2 (u_R).
  - Sensitivity on X_3 (γ_R) is less than heuristically expected.



LHS 6000 PCE4 256 PCE6 1296 ACOSSO 256 A-EXACT 160k R-EXACT 160k

LHS 60000 JRC 196k SDP 256 DACE 256 A-EXACT-2.56M R-EXACT-2.56M

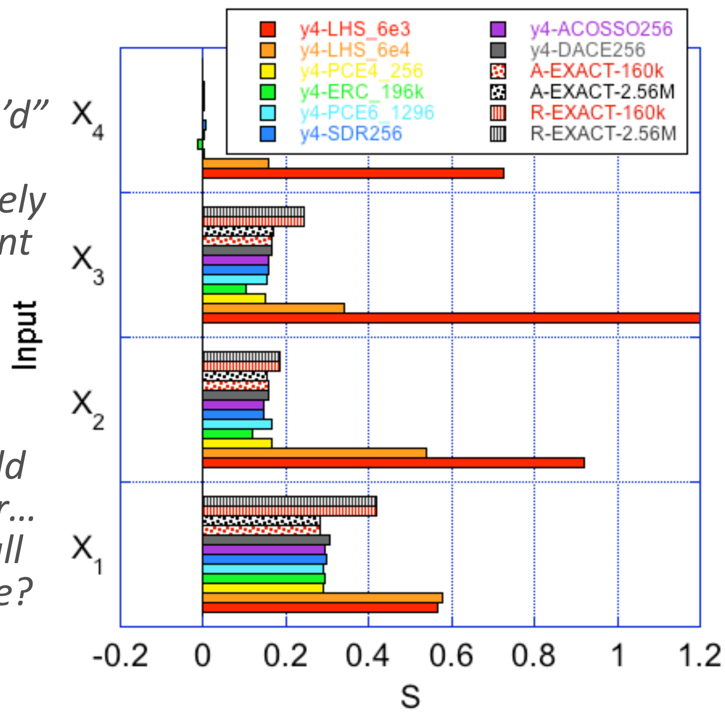
The sensitivity indices for Y_4 also show some unusual features.

- As expected, Y_4 (right $\Delta\rho$ time) depends strongly on X_1 (p_{R}).
  - Both small- and large-sample LHS values are quite different.

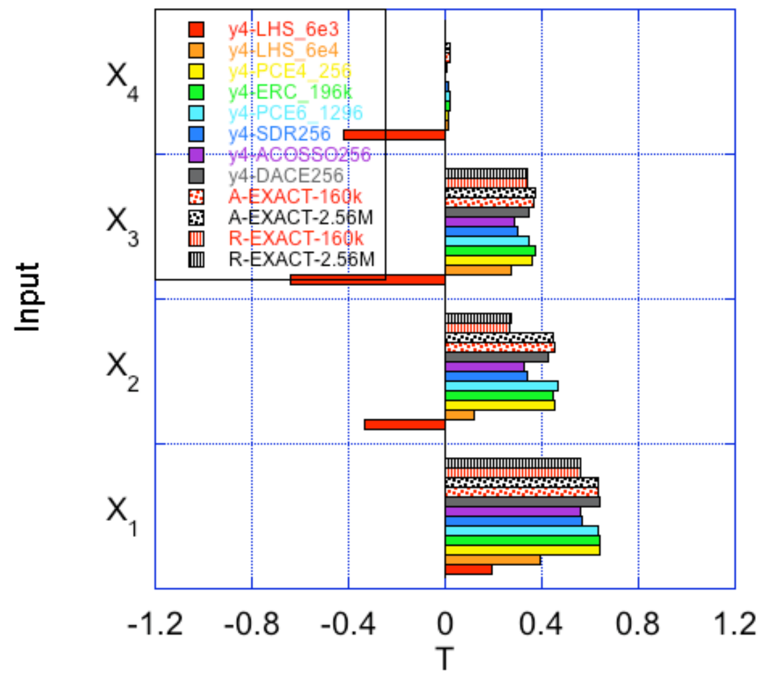
"Gold St'd" is not completely consistent

More odd behavior… Too small a sample?

Main



Total

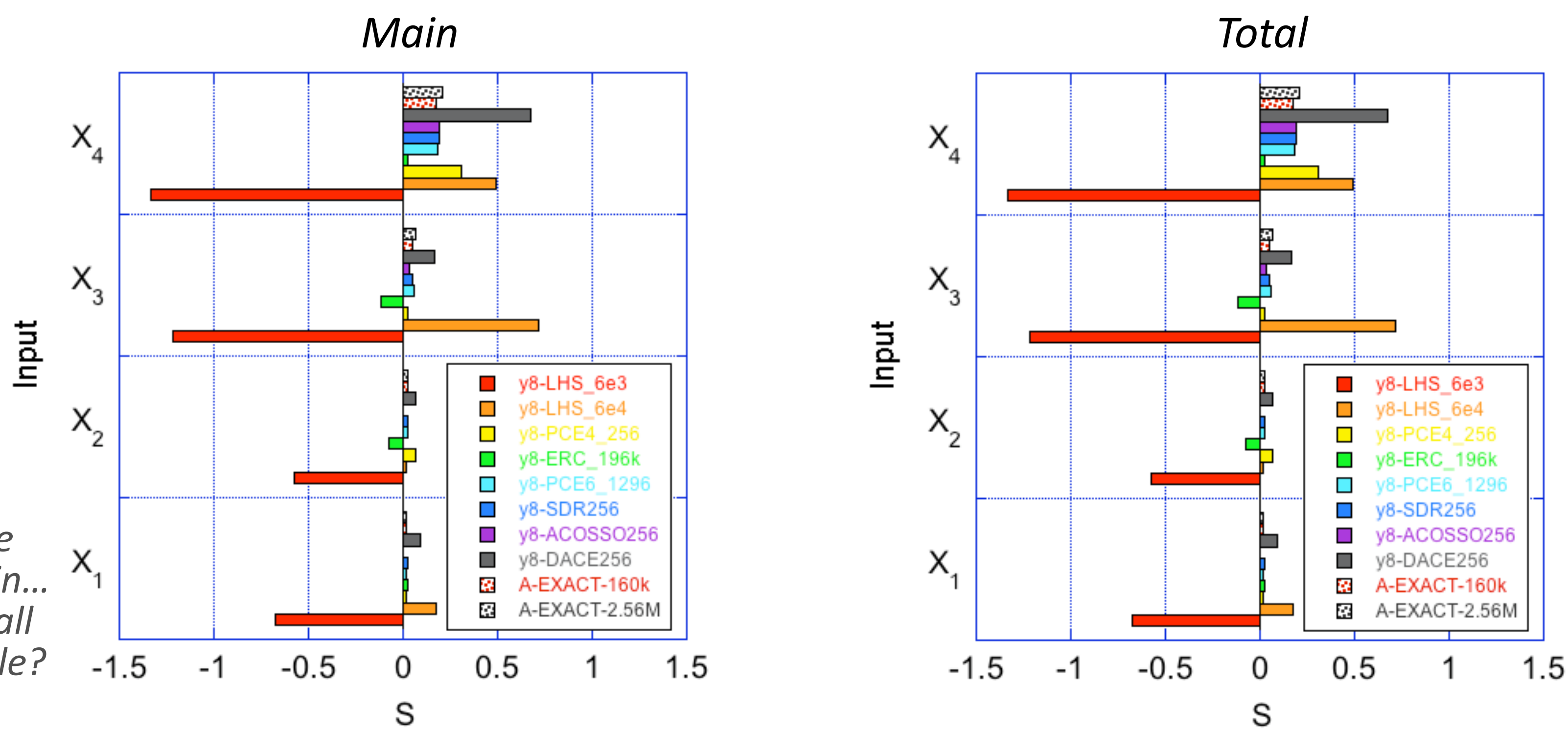


LHS 6000 PCE4 256 PCE6 1296 ACOSSO 256 A-EXACT 160k R-EXACT 160k

LHS 60000 JRC 196k SDP 256 DACE 256 A-EXACT-2.56M R-EXACT-2.56M

The total sensitivity index S for Y_8 exhibits some additional structure.

- Y_8 (CPU time) depends weakly on X_4 (CFL), indep. of others.
  - Total indices of different metamodels differ notably.



Here we go again… Too small a sample?

LHS 6000 PCE4 256 PCE6 1296 ACOSSO 256 A-EXACT 160k R-EXACT 160k

LHS 60000 JRC 196k SDP 256 DACE 256 A-EXACT-2.56M R-EXACT-2.56M

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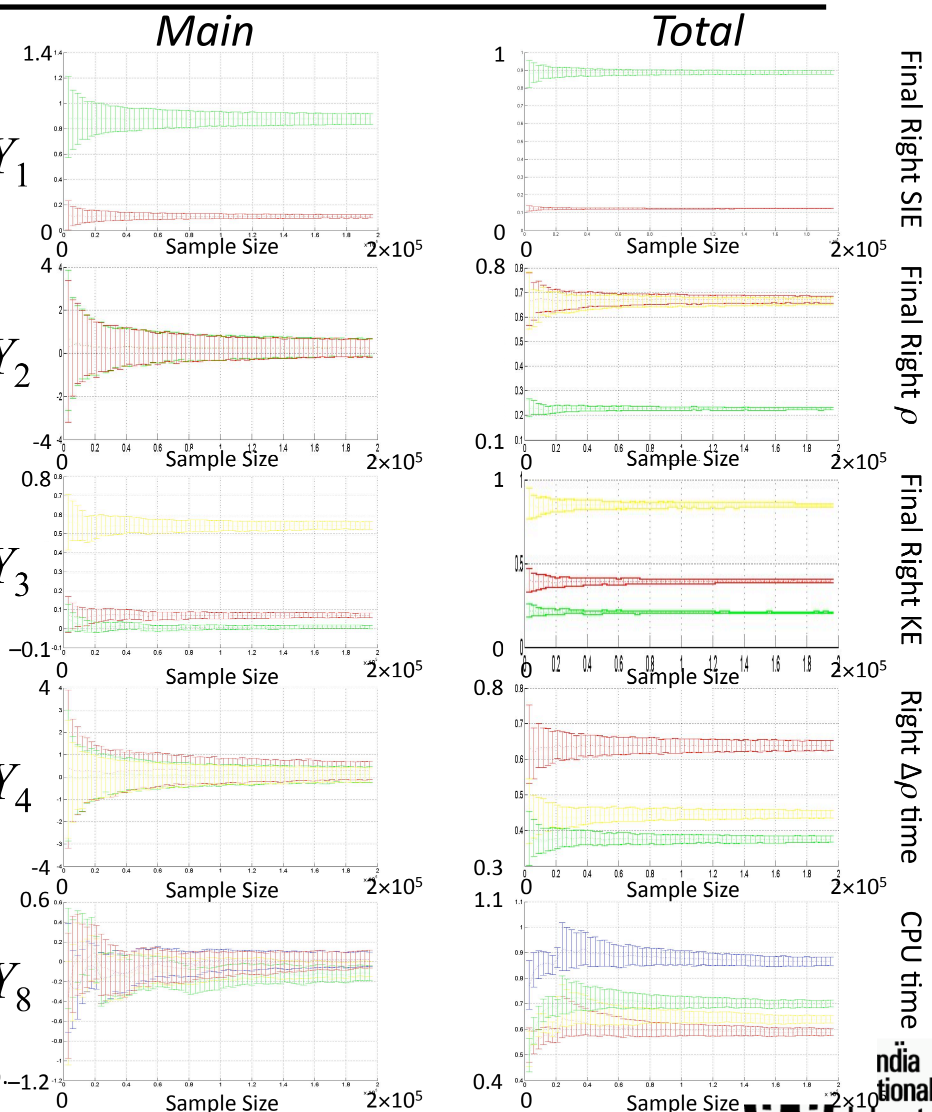
Estimators of the main and total sensitivity indices§ converge under quasi-random sampling.

- Confidence intervals obtained with bootstrap technique.*
- Confidence intervals decrease with increasing # of model runs.
- The lower/upper bounds of the main indices are wider than those of the total indices.
- The estimator of the main indices appears to have a larger variance than the estimator of the total indices.

X_1 Init. Right p	X_3 Init. Right γ
X_2 Init. Right u	X_4 CFL parameter



§ Saltelli, A., P. Annoni, I. Azzini, F. Campolongo, M. Ratto, S. Tarantola, "Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index," *Comp. Physics Comm.*, **181**, 259–270 (2010).

* G.E.B. Archer, A. Saltelli, I.M. Sobol´, "Sensitivity Measures, ANOVA-Like Techniques and the Use of Bootstrap," *J. Statist. Comput. Simul.*, **58**, pp. 99–120 (1997).

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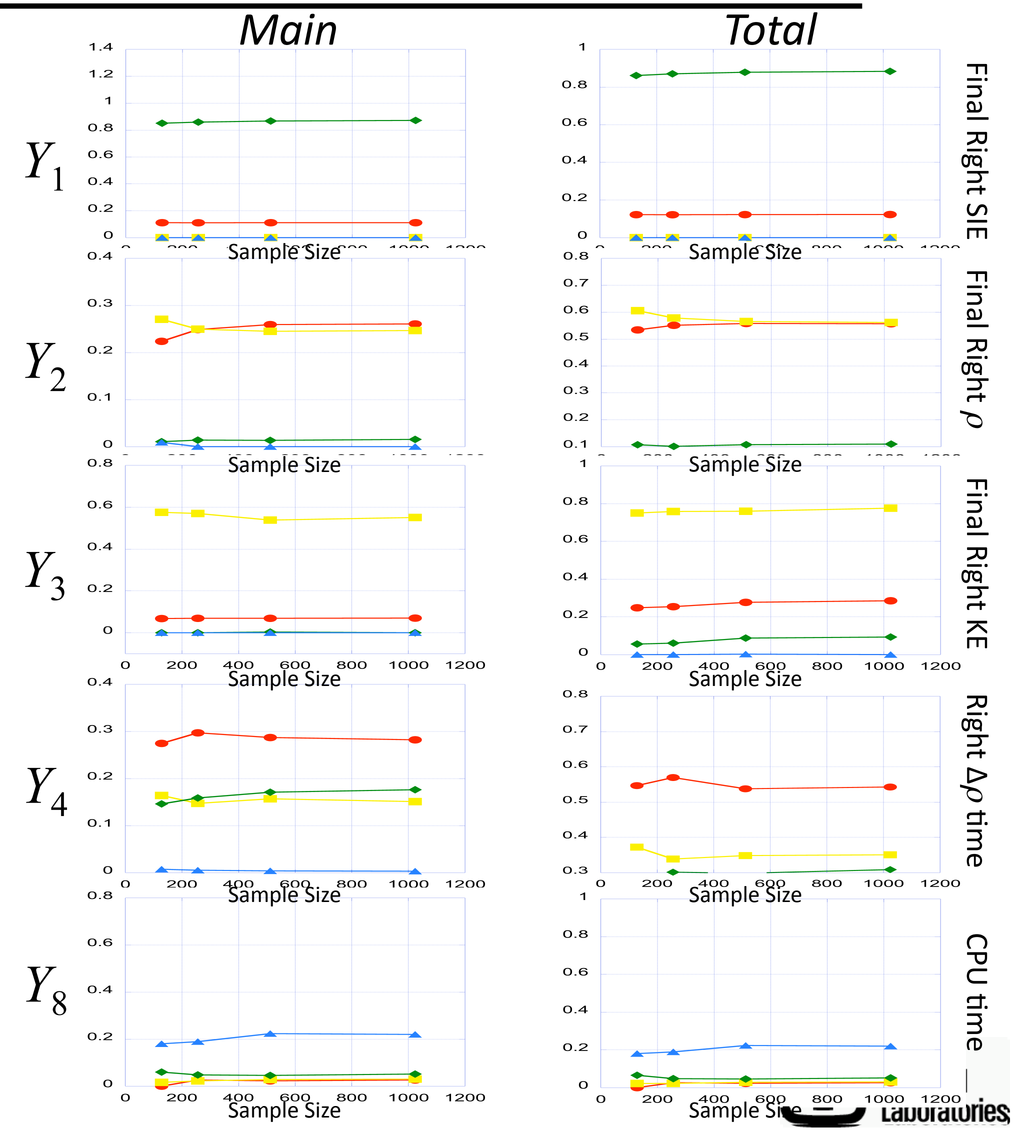
Variation of SDP results with sample size…

- Meta-model results are for SDP built with sample sizes: N=128, 256, 512, 1024.
- Sobol´ indices are estimated using only those runs that have been made to build the SDP meta-model.
- Both main and total indices are well-behaved from the perspective of convergence.
- Although we did not compute convergence rates, it appears that the indices from N=256 are already quite good.

X_1 Init. Right p	X_3 Init. Right γ
X_2 Init. Right u	X_4 CFL parameter



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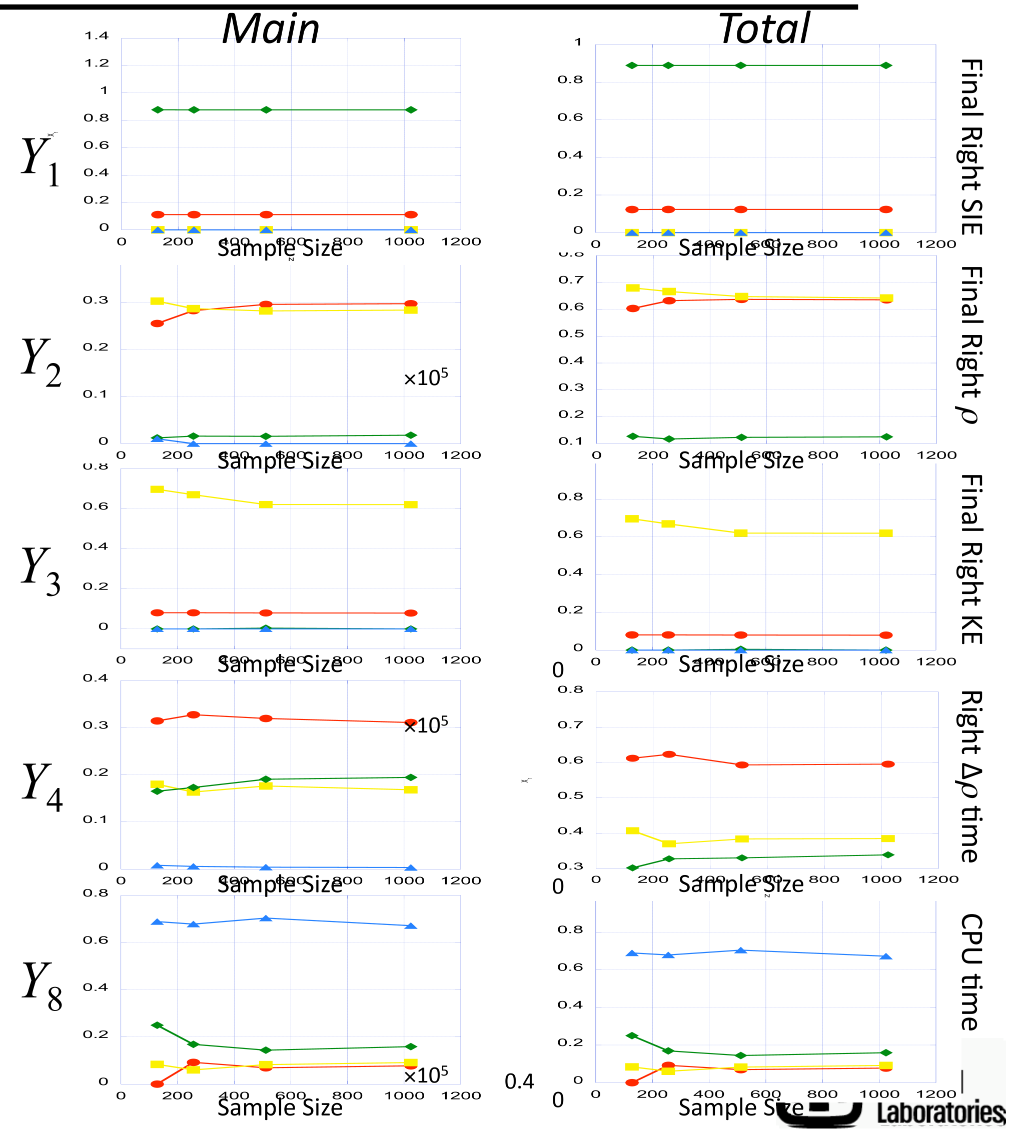
Variation of SDP + Sobol´ results with sample size…

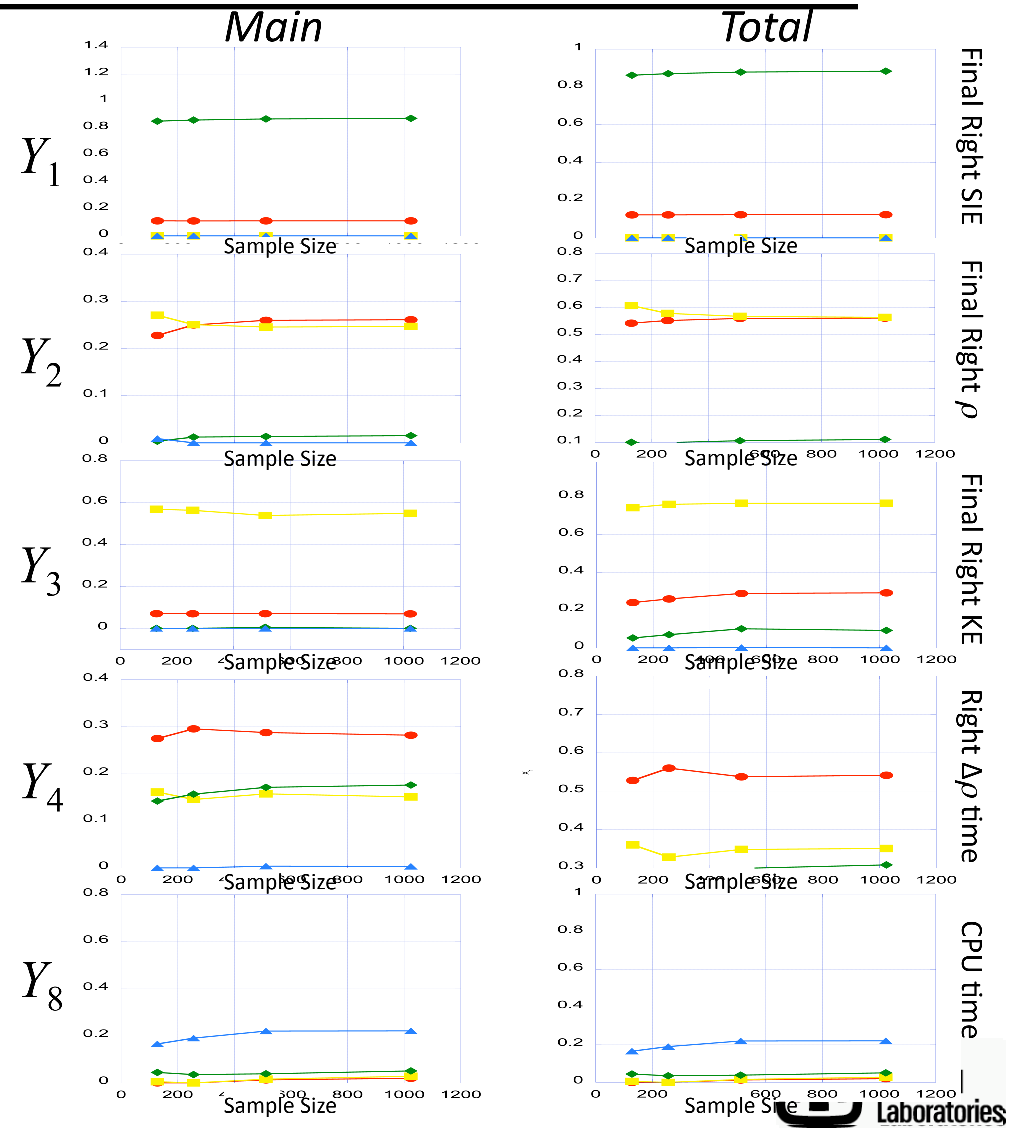
- Meta-model results are for SDP + Sobol´ esimators built with sample sizes: N=128, 256, 512, 1024.
- Sobol´ indices are are obtained executing the meta-model at a set of untried points.
- Both main and total indices are well-behaved from the perspective of convergence.
- Again, the indices from N=256 are already quite robust to further refinement.



X_1 Init. Right p	X_3 Init. Right γ
X_2 Init. Right u	X_4 CFL parameter

Variation of ACOSSO results with sample size....

- Meta-model results are for ACOSSO emulators built with sample sizes: N=128, 256, 512, 1024.
- Sobol´ indices are estimated using only those runs that have been made to build the ACOSSO meta-model.
- Again, the estimators of the main and total indices are well-behaved from the perspective of convergence.



X_1 Init. Right p	X_3 Init. Right γ
X_2 Init. Right u	X_4 CFL parameter

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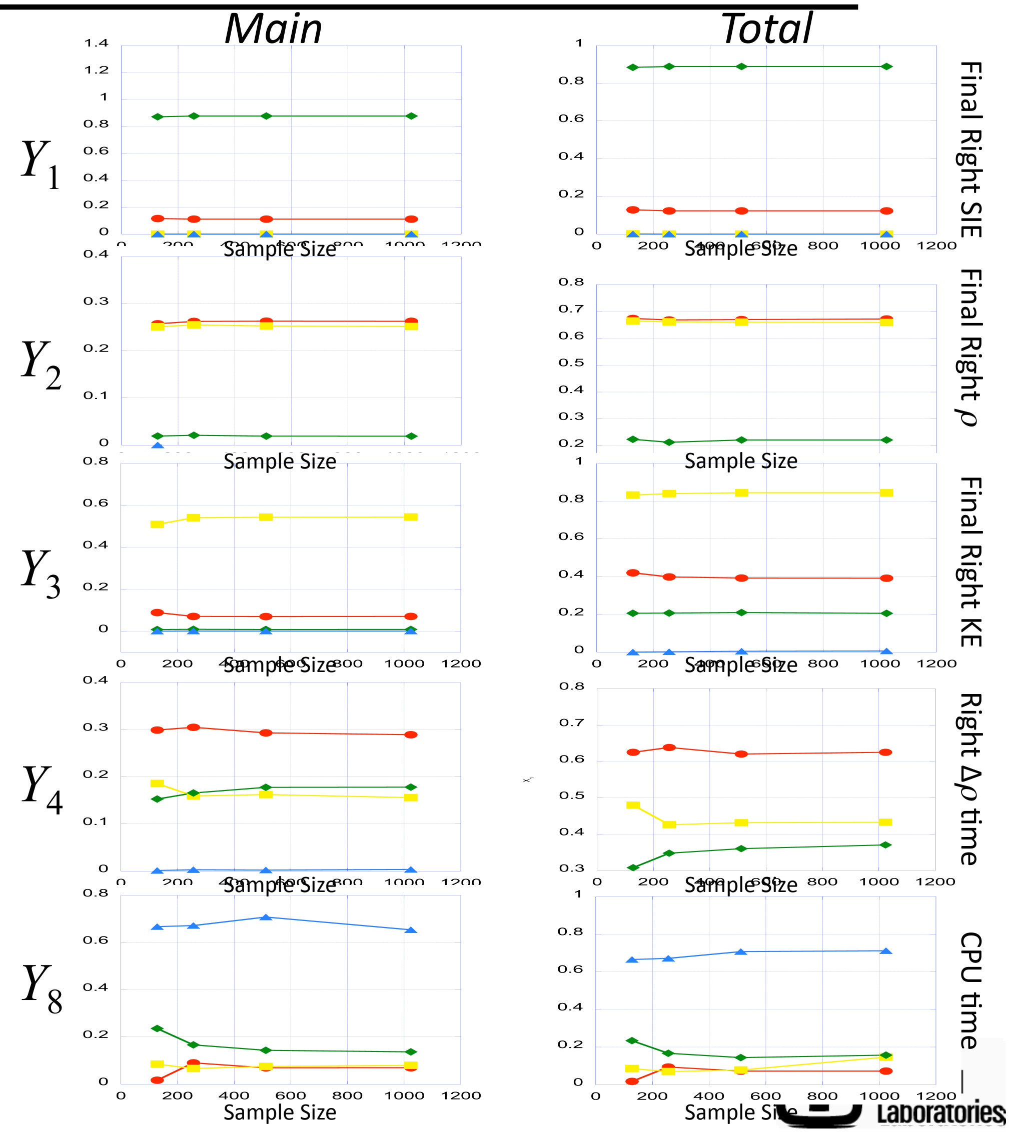
Variation of DACE + Sobol´ results with sample size…

- Meta-model results are for DACE + Sobol´ emulators built with sample sizes: N=128, 256, 512, 1024.
- Sobol´ indices are are obtained executing the meta-model at a set of untried points.
- The same story as the other meta-models, with respect to (heuristic) convergence.



X_1 Init. Right p	X_3 Init. Right γ
X_2 Init. Right u	X_4 CFL parameter

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We have some answers to our questions…

- *Do these approaches give consistent results, e.g., for rankings?*
  - In general, the different meta-models are consistent, both in ranking and magnitude, particularly for main effects (less so for total effects).
  - LHS sampling at 6k and 60k samples are often inconsistent with others.
- *Do these results vary for the different outputs?*
  - “Well-behaved” outputs (e.g., Y_1 and Y_3) are quite consistent.
  - “Less-well-behaved” output (Y_8) shows much greater variability.
- *How to these results depend on the different inputs?*
  - “Well-behaved” inputs (e.g., X_1, X_2) follow the above pattern.
  - Other inputs (X_3, X_4) show more variation for SDP and ACOSSO.
  - Correct index values can be more challenging to properly calculate when there are significant interactions among the inputs (e.g., Y_2)
- LHS (6000, in particular) is a fairly consistent outlier…

We have some answers to our questions…

- *Do these results "converge"?*
  - Yes (empirically): more samples → the results "settle down"
  - Yes and No: the "converged" value might differ from the exact value.
- *How to sampling and meta-model results compare?*
  - It varies: our LHS-based results sometimes looked "funny"…
- *Can we distinguish among different meta-models?*
  - The actual numbers varied slightly, but the rankings are robust.
- *How to exact solution results compare to ALEGRA results?*
  - "Well-behaved" inputs (e.g., X_1, X_2) follow the above pattern.

- LHS 6000
- LHS 60000

- PCE4 256
- JRC 196k
- PCE6 1296
- SDP 256
- ACOSSO 256
- DACE 256

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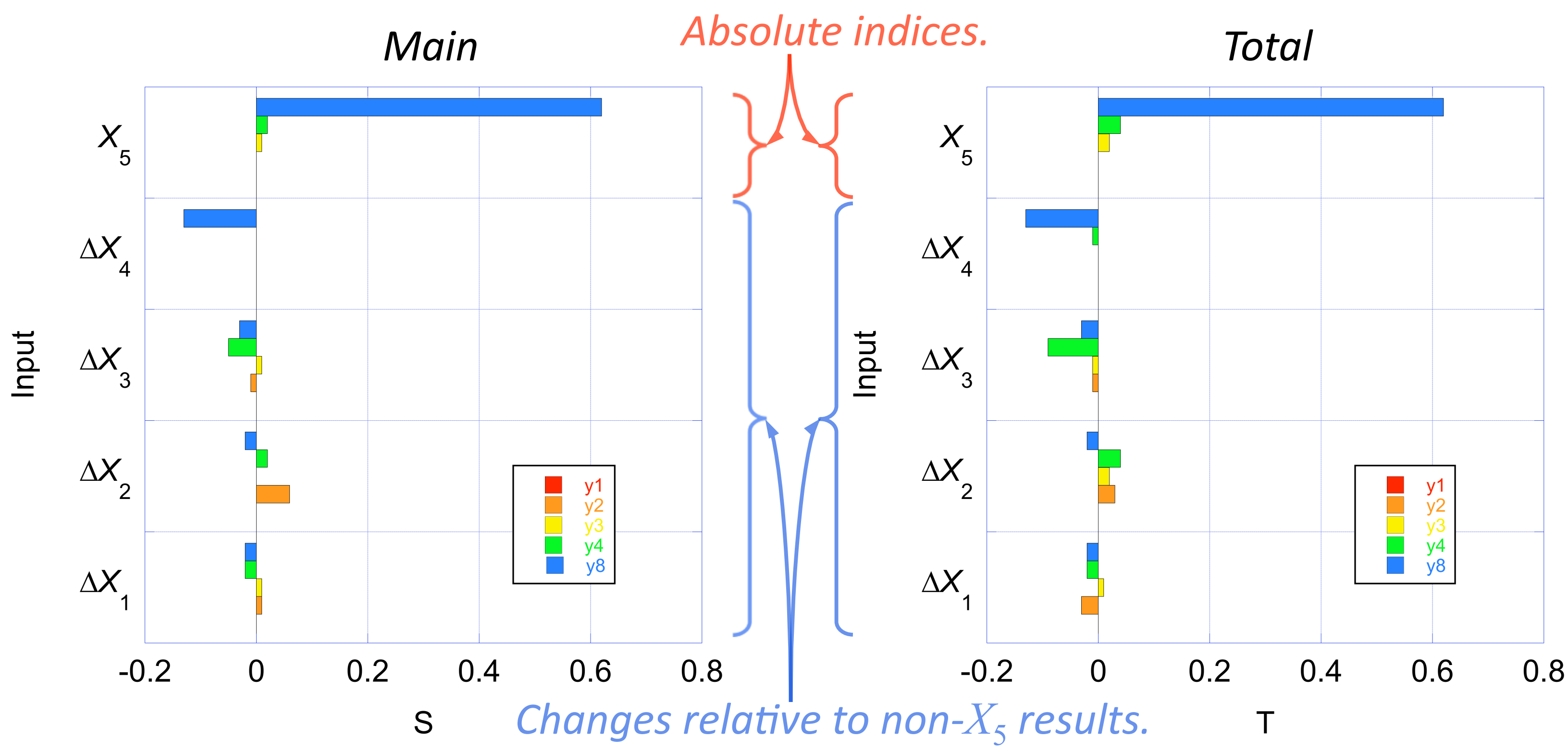
We also considered additional *discrete* inputs.

		Input	*Why?*
Continuous	X_1	Initial pressure on right	Uncertainty in initial condition
	X_2	Initial velocity on right	Uncertainty in initial condition
	X_3	Polytropic index γ on right	Uncertainty in material model
	X_4	CFL parameter: $c_s \Delta t / \Delta x$	Numerical parameter
Discrete	X_5	Time integration method	Different numerical algorithms
	X_6	Linear artificial viscosity	Different physical models

- Time integration algorithms are either 1st or 2nd order (2x CPU time).
- Linear artificial viscosity is either "off" (zero) or "on" (nominal value).
- Independent: $\{(X_1 - X_4) + X_5\}$ *or* $\{(X_1 - X_4) + X_6\}$, with N=128, 256, 512, 1024.
- We present results only of the SDP meta-model analysis.
- *Can we incorporate discrete + continuous? Effect on other sensitivities?*

Input X_5 (time integration scheme) affects Y_8 (CPU time) the most significantly for SDP 256.

- There is a noticeable change in the sensitivity of Y_8 to X_4 (CFL parameter, also related to the timestep), as well.

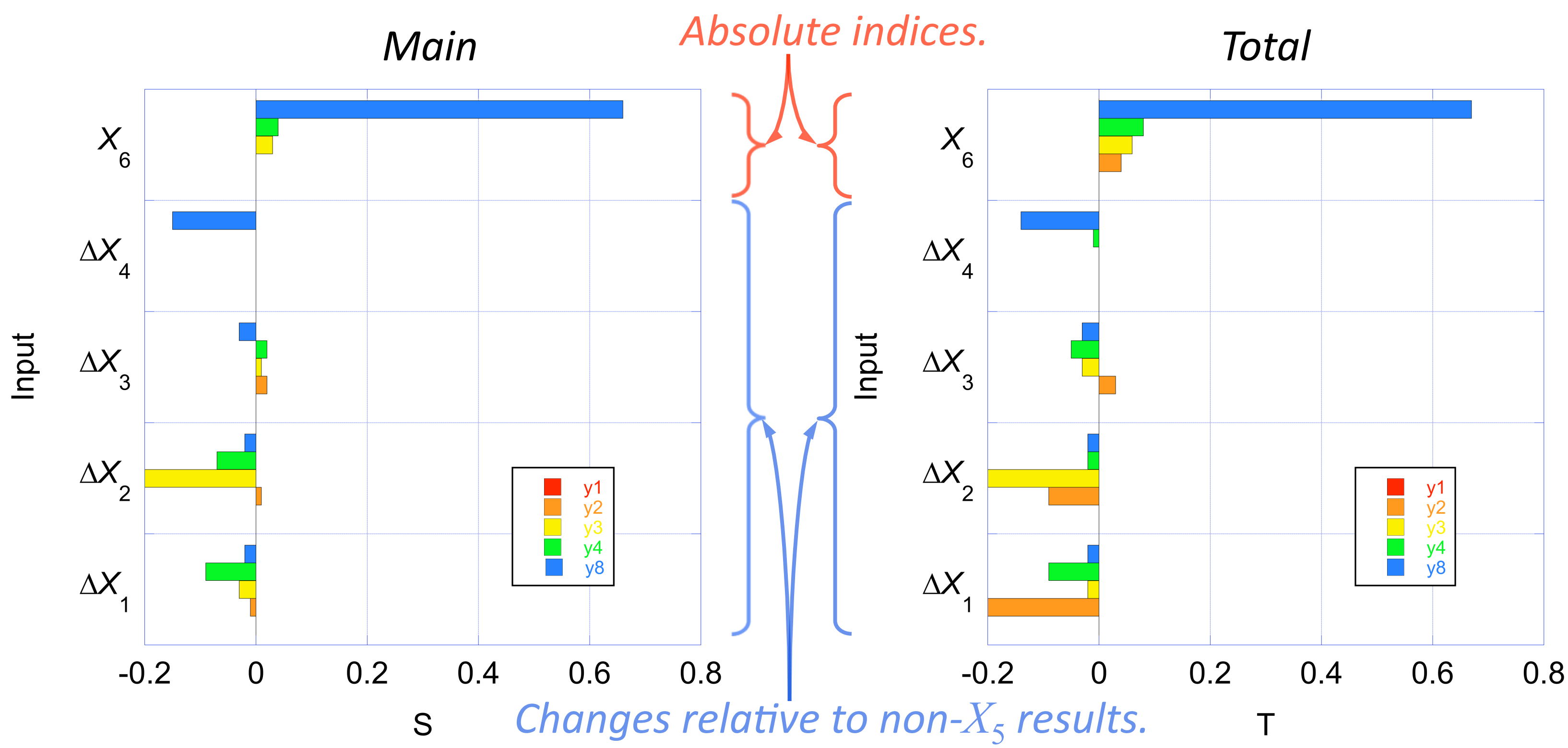


Y_1: SIE Y_2: Right ρ Y_3: Right KE Y_4: Right $\Delta\rho$ time Y_8: CPU time

There is identically zero effect of X_5 on Y_1

Input X_6 (artificial viscosity) also affects Y_8 (CPU time) the most significantly for SDP 256.

- There is a noticeable change in the sensitivity of Y_3 (final right KE) to X_2 (initial right velocity) in main and total indices.



Y_1: SIE Y_2: Right ρ Y_3: Right KE Y_4: Right $\Delta\rho$ time Y_5: CPU time

DACE $\rightarrow$ influence of X_6 on Y_2, Y_3, Y_4: inconsistent with other methods.

We have some additional answers…

- *Can we incorporate discrete + continuous?*
  - *Yes.* Just use repeated samples in constructing regression-type meta-models (SDP, ACOSSO, DACE).
  - It is still unclear, however, how to incorporate this into **PCE**…
- *What is the effect of these discrete inputs on the other computed results?*
  - Small, with one exception: the "less-well-behaved" output (Y_8)
- These results suggest that we can use the meta-modeling approach for SA of more complex physical models (e.g., different EOSs, material strength, fracture) involving both discrete and continuous inputs together.

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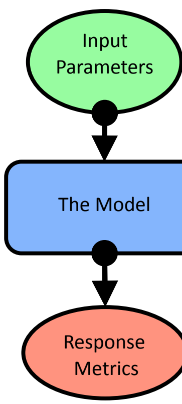
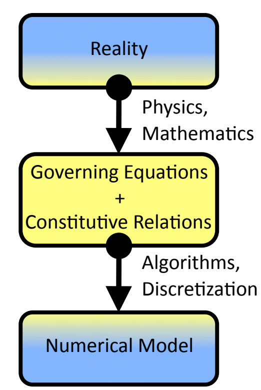
Summary: What did we talk about?

- The Sensitivity Analysis story:
  - Sobol'/Saltelli estimators of indices from quasi-random sampling
  - Sensitivity analysis using meta-models
    - PCE
    - SDP
    - ACOSSO
    - DACE (GP)



- The Application Space story:
  - The specific problem considered—and why
  - Inputs, outputs, and what we expected
- Computer Simulations:
  - The sensitivity analysis of the simulation model does not always match that of the exact model



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Conclusions: What we determined

- We considered real-physics test problem, with an exact sol'n.
- The response surface for computed and exact solutions could be compared and exhibited discontinuous behavior.
- LP-tau sampling gave bounded but non-unique convergence for standard sensitivity measures.
- All meta-models gave consistent main effects index values:
  - Comparable values – Comparable rankings – Converged
- Greater variability and differing rankings were seen for some outputs with both "small" and "large" LHS-based indices.
- We can incorporate discrete + continuous inputs.
- Differences between the computational model and the exact model were observed.

Abstract

Uncertainty quantification techniques are increasingly important in the interpretation of data and numerical simulations. Such techniques are typically employed either on data with poorly characterized underlying dynamics or on values from highly idealized model evaluations. We examine the application of these techniques to an intermediate case, in which data are generated from coupled, nonlinear partial differential equations—conservation laws—that admit discontinuous solutions. The values we analyze are generated from the numerical solution of the PDEs, in which we systematically vary both (i) fundamental modeling parameters and (ii) the underlying numerical algorithms. A number of sensitivity tests will be performed in order to assess the relative importance of such different types of uncertainty and we draw preliminary conclusions and speculate on the implications for more complex simulations.

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VBD Implementation $S_i = \frac{Var(E(Y \mid X_i))}{Var(Y)}$

- **Full Factorial:**
  - Take n values of each variable X_i, total samples are a full tensor product of n samples in each dimension. Total samples $N = n^d$.
  - For each particular value of X_i, calculate the average over the other X_j variables.

$$E(Y \mid X_i = x_{ik})$$

  - Calculate the variance of this expectation (variance over n values)

$$Var(E(Y \mid X_i))$$

- **Sampling approximation in *Sensitivity Analysis in Practice*, Saltelli et al. 2004:**
  - Calculate two independent sample matrices, A and B, with d (number of inputs) columns and n rows . C_i is constructed by taking the i^{th} column of A and substituting it into B.
  - Y_A, Y_B ,and Y_{Ci} are the vectors of responses from evaluating the simulator at the sample values in A, B, or C_i.
  - Total samples is (2+d)*n
  - Requires that n is of order of thousands for accuracy

$$f = \frac{Y_A \bullet Y_B}{N}$$

$$\textit{estimated}\ \mathrm{var}(Y) = \left(\frac{1}{N-1} Y_A \bullet Y_A\right) - f^2$$

$$S_i = \frac{\left(\frac{1}{N-1} Y_A \bullet Y_{c_i}\right) - f^2}{\textit{estimated}\ \mathrm{var}(Y)}$$

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The sensitivity indices S and T for Y_1 perform similarly for all approaches.

- As anticipated, Y_1 (SIE) depends strongly on X_1 (p_R) and X_3 (γ_R)

Main

	X_1	X_2	X_3	X_4
LHS_6e3	0.095	0.008	0.92	0.008
LHS_6e4	0.111	0.001	0.877	0.001
PCE4_256	0.1119	0	0.8764	0
JRC_196k	0.1116	0	0.8767	0
PCE6_1296	0.1116	0	0.8767	0
SDP256	0.1108	0	0.8595	0
ACOSSO256	0.1107	0	0.8597	0
DACE256	0.1116	0	0.8767	0

Total

	X_1	X_2	X_3	X_4
LHS_6e3	0.117	-0.014	0.864	-0.014
LHS_6e4	0.116	-0.003	0.886	-0.003
PCE4_256	0.1236	0	0.8881	0
JRC_196k	0.1233	0	0.8884	0
PCE6_1296	0.1233	0	0.8884	0
SDP256	0.1218	0	0.8706	0
ACOSSO256	0.1218	0	0.8707	0
DACE256	0.1233	0	0.8884	0

The sensitivity indices for Y_2 have some unusual features.

- For Y_2 (final right ρ), LHS has different ranking, particularly for 6.e+3 samples and esp. wrt X_3 (γ_R) and X_4 (CFL).

Main

	X_1	X_2	X_3	X_4
LHS_6e3	0.556	0.44	1.101	0.628
LHS_6e4	0.639	0.294	0.008	0.094
PCE4_256	0.2546	0.2554	0.0201	0.0001
JRC_196k	0.2578	0.2433	0.0204	-0.0011
PCE6_1296	0.2707	0.2439	0.0173	0.0001
SDP256	0.2489	0.2497	0.0139	0
ACOSSO256	0.2501	0.2508	0.0123	0
DACE256	0.2625	0.2551	0.0209	-0.0015

Total

	X_1	X_2	X_3	X_4
LHS_6e3	0.325	0.445	∞	-0.139
LHS_6e4	0.191	0.465	0.088	-0.217
PCE4_256	0.6673	0.6599	0.2321	0.0039
JRC_196k	0.6714	0.6636	0.2262	0.0049
PCE6_1296	0.6863	0.656	0.2163	0.0044
SDP256	0.5515	0.5787	0.1012	0
ACOSSO256	0.5521	0.5787	0.0989	0
DACE256	0.6682	0.6612	0.2132	0.0043

The sensitivity indices for Y_3 perform similarly for all approaches.

- As anticipated, Y_3 (final right KE) depends strongly on X_2 (u_R).
  - Sensitivity on X_3 (γ_R) is less than heuristically expected.

Main

	X_1	X_2	X_3	X_4
LHS_6e3	0.096	0.606	0.044	-0.003
LHS_6e4	0.07	0.537	-0.017	0
PCE4_256	0.0712	0.5409	0.0076	0.0005
JRC_196k	0.0706	0.5438	0.009	0.0005
PCE6_1296	0.0704	0.5417	0.0078	0.0005
SDP256	0.0693	0.5704	0	0
ACOSSO256	0.0702	0.5635	0	0
DACE256	0.0703	0.5414	0.0089	0.0003

Total

	X_1	X_2	X_3	X_4
LHS_6e3	0.318	0.787	0.145	-0.014
LHS_6e4	0.372	0.851	0.222	-0.004
PCE4_256	0.3955	0.8391	0.2038	0.006
JRC_196k	0.393	0.846	0.2058	0.0064
PCE6_1296	0.3943	0.8428	0.2071	0.0069
SDP256	0.2541	0.7586	0.0607	0
ACOSSO256	0.2595	0.7611	0.0698	0
DACE256	0.3989	0.8405	0.2073	0.0017

The sensitivity indices for Y_4 also show some unusual features.

- As expected, Y_4 (right $\Delta\rho$ time) depends strongly on X_1 (p_R).
  - Both small- and large-sample LHS values are quite different.

Main

	X_1	X_2	X_3	X_4
LHS_6e3	0.568	0.921	1.2	0.724
LHS_6e4	0.577	0.538	0.34	0.157
PCE4_256	0.2884	0.1671	0.1512	0.0022
JRC_196k	0.293	0.1181	0.1022	-0.0124
PCE6_1296	0.2888	0.1668	0.153	0.0028
SDP256	0.2972	0.1472	0.1588	0.0052
ACOSSO256	0.2956	0.1454	0.1569	0
DACE256	0.3051	0.1594	0.1658	0.003

Total

	X_1	X_2	X_3	X_4
LHS_6e3	0.193	-0.331	-6.43E-01	-0.42
LHS_6e4	0.391	0.121	0.274	0.013
PCE4_256	0.6393	0.4548	0.3594	0.0161
JRC_196k	0.6386	0.4458	0.3748	0.0192
PCE6_1296	0.6319	0.465	0.3476	0.0172
SDP256	0.5699	0.3387	0.3014	0.0107
ACOSSO256	0.5602	0.3283	0.2843	0
DACE256	0.6384	0.4262	0.348	0.0065

The total sensitivity index S for Y_8 exhibits some additional structure.

- Y_8 (CPU time) depends weakly on X_4 (CFL), indep. of others.
  - Both small- and large-sample LHS values are different.

Main

	X_1	X_2	X_3	X_4
LHS_6e3	-0.678	-0.577	-1.217	-1.33
LHS_6e4	0.177	0.014	0.719	0.49
PCE4_256	0.0145	0.0691	0.0221	0.3088
JRC_196k	0.0258	-0.0706	-0.1141	0.0255
PCE6_1296	0.0181	0.0244	0.0592	0.18
SDP256	0.0261	0.0224	0.0477	0.1892
ACOSSO256	0	0	0.0353	0.1905
DACE256	0.0905	0.0664	0.1666	0.6727

Total

	X_1	X_2	X_3	X_4
LHS_6e3	-0.764	-0.824	5.50E-02	0.85
LHS_6e4	0.991	1.176	0.519	0.923
PCE4_256	0.4352	0.5117	0.4682	0.7461
JRC_196k	0.5885	0.6389	0.7027	0.8681
PCE6_1296	0.5896	0.5829	0.6428	0.8041
SDP256	0.0261	0.0224	0.0477	0.1892
ACOSSO256	0	0	0.0353	0.1905
DACE256	0.0939	0.0697	0.1668	0.6713

Input X_5 (time integration scheme) affects Y_8 (CPU time) the most significantly for SDP 256.

- There is a noticeable change in the sensitivity of Y_8 to X_4 (CFL parameter, also related to the timestep), as well.

Main

	ΔX_1	ΔX_2	ΔX_3	ΔX_4	X_5
Y_1	0	0	0	0	0
Y_2	0.01	0.06	-0.01	0	0
Y_3	0.01	0	0.01	0	0.01
Y_4	-0.02	0.02	-0.05	0	0.02
Y_8	-0.02	-0.02	-0.03	-0.13	0.62

Total

	ΔX_1	ΔX_2	ΔX_3	ΔX_4	X_5
Y_1	0	0	0	0	0
Y_2	-0.03	0.03	-0.01	0	0
Y_3	0.01	0.02	-0.01	0	0.02
Y_4	-0.02	0.04	-0.09	-0.01	0.04
Y_8	-0.02	-0.02	-0.03	-0.13	0.62

Changes relative to non-X_5 results. *Absolute indices.*

  - There is identically zero effect of X_5 on Y_1
  - **Say something else**

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Curiously, input X_6 (artificial viscosity) also affects Y_8 (CPU time) the most significantly for SDP 256.

- There is a noticeable change in the sensitivity of Y_3 (final right KE) to X_2 (initial right velocity) in main and total indices.

Main

	ΔX_1	ΔX_2	ΔX_3	ΔX_4	X_5
Y_1	0	0	0	0	0
Y_2	-0.01	0.01	0.02	0	0
Y_3	-0.03	-0.2	0.01	0	0.03
Y_4	-0.09	-0.07	0.02	0	0.04
Y_8	-0.02	-0.02	-0.03	-0.15	0.66

Total

	ΔX_1	ΔX_2	ΔX_3	ΔX_4	X_5
Y_1	0	0	0	0	0
Y_2	-0.21	-0.09	0.03	0	0.04
Y_3	-0.02	-0.21	-0.03	0	0.06
Y_4	-0.09	-0.02	-0.05	-0.01	0.08
Y_8	-0.02	-0.02	-0.03	-0.14	0.67

Changes relative to non-X_5 results. *Absolute indices.*

- DACE $\rightarrow$ influence of X_6 on Y_2, Y_3, Y_4; other methods do not confirm this.
- There is identically zero effect of X_6 on Y_1